Distributed output regulation of multi-agent systems

Yiguang Hong

Academy of Mathematics and Systems Science
Chinese Academy of Sciences

Beijing, May 2012
Acknowledgments

Joint work with: Xiaoli Wang, Jie Huang, Zhong-Ping Jiang, Jiangping Hu, Dabo Xu, ...
Outline

- Introduction
- Formulation
- Main Results
- Conclusions
1. Multi-agent system (MAS)

Agent: subsystem with sensors and actuators

In Nature: particles, neural network, ecological system, metabolic system...

In Society: urban development, economics, transportation network...

In Engineering: power grid, wired/wireless communication, sensor network, robotic network, software system...
Consensus: a basic problem

Agent dynamics: \( dx_i/dt = u_i \quad i = 1, \ldots, 2 \)
Leader (or desired position): \( x_0 \)

Neighbor-based communication \((N_i: \text{the neighbor set of agent } i)\)

Distributed control: \( u_i = \sum_j (x_j - x_i), j \in N_i \)

Multi-agent consensus (agreement, synchronization):
- **Leader-following**: \( x_i - x_0 \to 0 \)
- **Leaderless**: \( x_i - x_j \to 0 \)
Interaction Graph

- **Nodes**: agents, individuals, units, ...
- **Links**: ties, relationships, communications, ...
- **Weights on links**: value (strength, significance) of tie

Directed & undirected graphs: maybe time-varying
Multi-agent systems: distributed estimation and control (with constraints) to achieve collective tasks.

Problems (beyond consensus): formation, flocking, location search/rescue, ...

Concerns: connectivity (fixed or switched), coverage (dynamic or static), communication (bandwidth, delay), computation (real time, complexity)...

→ distributed output regulation
Output regulation

Output regulation: $e \rightarrow 0$

Stabilization, asymptotical tracking, disturbance rejection …
Internal model (IM)

Output regulation (OR): tracking between two different systems

**IM principle:**
incorporate a model of the exosystem to follow it

IM provides a systematical way for the (robust) output regulation
Results on output regulation

- **Linear systems** (Davison, Wonham, Francis, ...): classic IM (1970’s)
- **Nonlinear systems** (Isidori, Byrnes, Huang, ...): Modern IMs incorporate a model determined jointly by both the plant and exosystem (after 1990).
- **Large-scale systems** (Ding, Gazi, ...): decentralized control each agent can get the information of the exosystem
2 Formulation

Distributed output regulation (DOR): a framework for leader-follower coordination (consensus)

- **Exosystem** $\rightarrow$ active leader …
- **Dynamics** different from those of agents
- **Unmeasurable** states (internal dynamics)

**Plant** $\rightarrow$ **agent** $\rightarrow$ a group of follower-agents with (time-varying) interconnection topology
Decentralized vs. Distributed

Decentralized OR

Exosystem

agent1
control1

agent2
control2

agent3
control3

Distributed OR

Exosystem

agent1

agent2

agent3

agent4

agent5
Background for DOR:
- Location estimation of a moving target in a sensor network;
- Decentralized OR control by adding neighbor information;
- Following a virtual leader or reference trajectory;
- Synchronization with rejecting exogenous disturbance
Multiple Agents

Distributed OR:
Agent dynamics +
distributed rules

Exosystems: target, leader, disturbance ...
Multiple Agents

Distributed OR:
Agent dynamics +
distributed rules

Exosystems: target,
leader, disturbance …
Fundamental challenges

Linear systems:
- Solvability: when the output regulation of agents can be achieved using local neighbor information
- Design: how to extend IM-based design to DOR

Nonlinear systems:
- Solvability: necessary/sufficient conditions?
- Design: new IM? new interaction structure?
3. Main Results

- Simple design for simple systems with distributed dynamic feedback (Hong et al, Automatica 2006; Hong et al, Automatica 2008; Hong et al, JSSC 2009)

- “Universal” design based on IM (Wang, Hong et al, IEEE TAC 2010, Su, Hong et al, ICCA 2011; Hong et al, Int J Robust & Nonlinear Control 2012)

- Multi-graph design based on networked IM for nonlinear agents (Xu & Hong, CCC 2012)
Basics

Output regulation: Stability + solvability of regulator equation (RE) (which describes the constraint between two different systems)

DOR with multi-agent topology: (1) fixed and directed graph, (2) switching and undirected graph

\(\Rightarrow\) solution to switching RE

DOR control design: construction of distributed feedback based on neighbor information
3.1 Simple cases

First-order/second-order agents track an active leader without using IM:

1. Switching inter-agent topology keeps connected \( \Rightarrow \) target (leader) is tracked.

2. Bounded uncertainties + duration when the topology keeps connected is long \( \Rightarrow \) bounded tracking error.
3.2 IM-based design

Basic model:

Exosystem:
\[ \frac{dw}{dt} = \Gamma w; \quad w \in \mathbb{R}^k, \quad y_0 = Fw \in \mathbb{R}^l. \]

Agent model:
\[ \frac{dx_i}{dt} = Ax_i + Bu_i + Dw, \quad x_i \in \mathbb{R}^m, \quad y_i = Cx_i \in \mathbb{R}^l, \quad i = 1, \ldots, n \]

Control aim: \[ e_i = y_i - y_0 \rightarrow 0 \text{ as } t \rightarrow \infty \]
1. Connectivity of directed graph: exosystem (node 0) is globally reachable
2. \((A, B)\) is stabilizable (and \((C, A)\) is detectable)
3. Real parts of eigenvalues of the exosystem are nonnegative \((\sigma(\Gamma) \geq 0)\), to avoid trivial discussion
4. Rank condition: \(\Lambda(\Gamma)\) spectrum of \(\Gamma\)

\[
\operatorname{rank} \begin{pmatrix} A - \lambda I & B \\ C & 0 \end{pmatrix} = n + q, \quad \lambda \in \Lambda(\Gamma)
\]
Difficulties

1. Distributed design: neighbor-based rule, not completely-connected
2. Unmeasurable variables of leader (target): dynamic control with online estimation → solvability of distributed regulator equation (RE) and stability analysis
3. Switching structure: common Lyapunov function
4. Effective control design
Neighbor-based control:

- **Static feedback (not IM-based):**
  \[ u_i = K_z z_i + K_x x_i \]

- **Dynamic feedback:** (state or output)
  \[
  \begin{align*}
  u_i &= K_z z_i + K_x x_i + K_v v_i \\
  \dot{v}_i &= E_z z_i + E_x x_i + E_v v_i
  \end{align*}
  \]

- Relative error \( z_i \) based on \( N_i \) (neighbor set of agent \( i \)), for example:
  \[
  z_i = \sum_{j \in N_i} a_{ij} (x_i - x_j) + 1_{(i \in N_0)} a_{i0} (x_i - C^+ F w)
  \]
Closed loop system

\begin{align*}
\dot{\xi} &= A_c \xi + B_c w \\
\dot{w} &= \Gamma w \\
e &= (I_N \otimes C_x)x - (1 \otimes F)w
\end{align*}

\( x=(x_1, \ldots x_n) : \text{state} \)

\( \xi=x \) in static feedback

\( \xi=(x, v) \) in dynamic feedback

Regulation constraints (equivalent to RE)

\begin{align*}
X_c \Gamma &= A_c X_c + B_c \\
C_c X_c &= F_c = 1 \otimes F
\end{align*}
Internal Model (IM) approach: output regulation \rightarrow stability.

Two steps in analysis of distributed IM-based control:

- Step 1: transformation based on graph of the network topology \rightarrow “virtual agent”
- Step 2: simultaneous control of “virtual agents”
Two more steps for OR with switching topology

- Step 3: check and find common regulation matrix $X_c$ as the solution for switching regulator equations
- Step 4: find common Lyapunov function $\Rightarrow$ distributed design to achieve DOR
Main results

“Universal” design based on IM in the compensator: systematical and robust

- Fixed topology: necessary and sufficient condition of the solvability of DOR (that is, the rank condition); feedback design for different cases

- Switching topology: dynamic feedback with more neighbor information
Nonlinear systems?

Extended to nonlinear multi-agent systems?
- Every agent is connected to the leader (exosystem)
- Tree structures for special nonlinear systems
- Linearization → local results
3.3 Nonlinear DOR

Challenges in DOR design for multiple nonlinear agents:
1. Nonlinear OR is hard to be solved (solvability of nonlinear RE and nonlinear stabilization)
2. Limitation of decentralized control design
3. Exosystem: reference & disturbance → different ways to pass the information
4. Neighbor-based nonlinear control design
Effective design for DOR? Based on 2-level graph (cyber-physical systems):

1. Plant graph: physical connection between agents & measurement information
2. Controller graph: cyber connection between controllers
DOR Design to share the information of neighbor IM-based controllers.

**IM → networked IM:**

\[
\dot{\eta}_i = \alpha_i(\eta_i) + L_i(u_i + \lambda_i(\eta_i^b) - \beta_i(\eta_i))
\]

where \( \eta_i^b = (\eta_j, j \in \mathcal{O}_i^c) \)

The controller: \( u_i = h_i(e_i, \eta_i, \eta_j) \)
Results

With more information exchanges, solve the unsolvable problems in decentralized control, and reduce the complexity of IM-based controller. There are solution of RE and stable generator ... → networked IM can be constructed to be input-to-state stable to solve DOR for fixed graphs without loop. Many remain to be done!
Example

The exosystem consists of two parts: leader & disturbance

- **Leader:** Lienard system
  \[ y_0 : \text{output of the leader} \]

- **Disturbance model:**
  \[
  \begin{align*}
  \dot{u}_r &= -G(u_{r2}), \\
  \dot{u}_{r2} &= v_{r1} - F(u_{r2}) \\
  y_0 &= v_{r1}
  \end{align*}
  \]

  \[
  \begin{align*}
  \dot{u}_d &= v_d, \\
  \dot{v}_d &= -\omega^2 u_d \\
  \dot{u}_d &= 0
  \end{align*}
  \]
Followers and plant graph

Agent 1 with $x_1$
\[
\begin{align*}
\dot{x}_1 &= u_1, \\
\xi_1 &= x_1 - y_0
\end{align*}
\]

Agent 2 with $x_2$
\[
\begin{align*}
\dot{x}_2 &= u_2 + x_1^2 - v_{d1}, \\
\xi_2 &= x_2 - x_1.
\end{align*}
\]

Agent 3 with $x_3 = \text{col}(z_3, y_3)$
\[
\begin{align*}
\dot{z}_3 &= -x_3 + \xi_3 y_3, \\
\dot{y}_3 &= u_3 - u_{d3} v_{d1} + x_2^2 + v_{d2}^3 - \sin^2(x_2) \\
\xi_3 &= y_3 - x_2.
\end{align*}
\]

Leader as node 0

Relative measurement $e_i$
Controller graph

Given a controller graph:

Cooperative controller based on networked IM: (cyber) IM variable $\eta_i$ exchanged over the controller graph

- Helpful to solve RE
- Largely reduce structural complexity of IMs for agents 2 and 3,...
4 Conclusions

- MAS: collective dynamics and distributed algorithms
- Distributed output regulation: framework and fundamental problems
- Challenges: nonlinear, stochastic, hybrid (event-triggered), ...
- MAS: cooperative control → cooperation in research
Thank you!